Kuwait University | Math.102 | 14 October 1999

Dept of Math &Comp Sci | First Midterm | Duration: 75 min

Answer all the following questions. Calculators and mobile phones are *not* allowed. Each (sub)question carries 5 marks.

1. Show that the function $f(x) = \sqrt{3 + e^{x-1}}$, $x \in [1, \infty)$ is one-to-one. Find $f^{-1}(x)$ and state its domain and range.

2. Find the slope of the tangent line to the graph of f^{-1} at the point $(3, 1)$, where $f(x) = x^7 + 3x^5 + x - 2$

3. Write $\tan(\sin^{-1} 3x)$ as an algebraic expression of x.

4. (a) Use logarithmic differentiation to find y' if $y = \dfrac{(x + \log_5 x)^2 (x^3 + 2x)^9}{\sqrt[3]{x^3 + 1}}$.

   (b) Use implicit differentiation to find y' if $x^4 + 2^{xy} - y^2 = 1$.

5. Evaluate

   (a) $\displaystyle\int_0^{\pi/4} \frac{2^{\tan x}}{\cos^2 x}\, dx$;

   (b) $\displaystyle\int \frac{\operatorname{sech}^2 x\, dx}{1 + 3\tanh x}$;

   (c) $\displaystyle\int \frac{dx}{\sqrt{e^{2x} - 4}}$;

   (d) $\displaystyle\int \frac{dx}{x(\ln x)^2}$.

6. Find $\displaystyle\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^2}$.